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4 pages

# Radiation-damping and the Betatron.

By BRUNO TOUSCHKE.

In a short notice in the Physical Review Iwanenko and Pomeranohuk<sup>u</sup> have pointed out that the effect of radiation damping due to the centripetally accelerated motion of the electrons would define an upper limit for the energy obtainable with the betatron. The genuine publication which appeared in a Russian periodical was not available according to the war, which made a complete reinvestigation by the author necessary.

The statement of I. and P. seemed at first sight objectionable by the two following considerations: 1) I. and P. had calculated as if there was only one electron in the betatron thus neglecting the possible influence of the other electrons on the energy-balance. 2) The radiation emitted is locked in the toroid-tube, which in general is covered by a thin metallic layer, thick enough to totally reflect any radiation with the wavelength of an order of magnitude of about 1 meter. Objection 1) may easily be disproved. Instead of the electrons revolving on the equilibrium circle we may as well consider an assembly of electronic oscillators which for simplicity may have unrelativistic energies. The oscillators may all have x-direction and the same initial energy, while their phases be distributed at random. Then the energy radiated in a certain direction is proportional to  $\ddot{Z}^2$  where Z is the Hertzian vector of the system defined by  $Z = e \sum_k x_k$ .  $x_k$  means the coordinate of the kth oscillator, which is a harmonic function of time. Now  $\ddot{Z}^2 = 2 \sum_k \ddot{x}_k \ddot{x}_k$ . When carrying out the time-average the mixed terms  $i \neq k$  disappear, so that finally the radiation actually emitted is proportional to  $n \ddot{x}^2$ , when n is the number of the oscillators concerned. By generalising this result, we see, that each electron radiates as if unperturbed by the others. The effect seems to be very similar

to the shot-effect of thermions which is also due to the fluctuation of phases.

Objection 2) however is as yet not disproved. One might as a model consider an oscillator in a box with totally reflecting walls. Since the oscillator represents a system with an absorption coefficient  $\neq 0$  for all frequencies (according to radiation damping) the final state of box+oscillator will undoubtedly be a black radiation field. The temperature of this radiation will be defined by the initial <sup>energy</sup> temperature of the oscillator. Since the oscillator has got one degree of freedom only, whilst the number of degrees of freedom of the radiation field is practically infinity, the oscillator will have lost all its energy in the final state. It is thus certain that the walls of the tube will not prevent energy to be lost, but as yet we have not derived the order of magnitude of the time during which the exchange of energy between oscillator and radiation-field takes place. There are two other questions arising out of this consideration: how does this time depend on the shape of the box and how does it depend upon the presence of the other electrons?

Let us pessimistically assume that objection 2) was settled in a negative sense and that the walls made no difference whatsoever. Then the result of the classical calculations can be written in the form

$$(1) \quad \Delta \dot{\gamma} = \frac{1}{3} \frac{v^2}{R^2} \gamma^4$$

Here  $\gamma$  means the energy measured with the rest-energy (0.511MeV) of the electron as a unit,  $v$  the velocity of light,  $r_e$  the classical electron-radius ( $2.8 \cdot 10^{-13}$  cm) and  $R$  the radius of the orbit of the uniformly revolving electrons.  $\Delta \dot{\gamma}$  is the energy-loss due to radiation, the point indicates differentiation with respect to time. At the betatron this energy-loss must be overbalanced by a gain of energy

Due to the accelerating effect of the alternating flux within the orbit. This gain of energy has in the extremely relativistic limit the form

$$(2) \quad \dot{\gamma} = \left( \frac{1}{2} \right) \gamma_m \omega \sin \omega t \quad \begin{matrix} 0 < \omega t < \frac{\pi}{2} \\ -\frac{\pi}{2} < \omega t < \frac{\pi}{2} \end{matrix}$$

$\gamma_m$  here means the naively (omitting the influence of radiation-damping) attempted maximum energy,  $\omega/2R$  the frequency of the accelerating field. The upper expression in brackets refers to the normal case, when a quarter of the period is taken for acceleration. According to an idea of W i d e r ö e however it is also possible to accelerate during one half of the period. This is made possible by superimposing a constant magnetic field in the neighbourhood of the electron orbit. The lower term in brackets refers to this case. The intern terminology of the Wideröe staff ~~agrees~~ uses the term: complete D-case. The complete D-case allows for duplication of energy and is not to be realised because of injection difficulties. The incomplete D-case takes place if any energy between  $2U_0$  and 0 is achieved with a betatron originally designed for a tension  $U_0$ .

The differential equation ruling the energy-balance of the betatron thus becomes

$$(3) \quad \dot{\gamma} = \left( \frac{1}{2} \right) \gamma_m \omega \sin \omega t - \frac{1}{3} \frac{c^2}{R^2} \gamma^4$$

Assuming the influence of the radiation damping to be small its effect may - according to (3) appear in two different forms: a) The decrease of energy implies a decrease of the radius  $R$  of the electronic orbits, since the centrifugal forces grow smaller than the L o r e n t z - force. This decrease of radius is roughly given by

$$(4) \quad 2 \frac{\delta R}{R} = \frac{\delta \gamma}{\gamma} \quad \delta \gamma = \gamma - \gamma_0 < 0$$

Since the tube has got a finite extension, the electrons will because of this effect hit the inner wall of the tube. This will take place if  $\delta R > \Delta R$ , when  $\Delta R$  is the distance between the equilibrium-orbit and the inner wall. b.) Then acceleration term is positive during the acceleration period. The radiation-damping adds a negative term



effects  $\gamma$  vanish and become negative earlier than it should. It turns out, that this influence may be neglected against the influence of a). It must be taken into account however, that it seems possible to overcome the difficulty a) to a certain degree by technical devices, while b) seems to be a complete apory. In their l.c. notice P. and I. have only considered b) which made their results rather optimistic.

Assuming the radiation-damping to be small an approximate integration of (3) becomes possible (replacing  $\gamma$  by  $\gamma_0$  in the radiation term). The result of this integration can best be expressed as a condition for the minimum frequency depending on the other details of the design. One easily obtains

$$(5) \quad f = \left( \frac{B}{\Delta R} \right)^{1/2} 10^{-3} \frac{B \gamma_{\text{rel}}}{\Delta R}$$

Here  $f$  is the frequency ( $\omega/2\pi$ ) of the betatron,  $\Delta R$  the maximum radius-shift admissible and  $B$  the maximum magnetic induction with 10.000 Gauss as a unit. It will be noted, that this minimum frequency is approximately 3 times smaller in the Normal case than in the complete D-case. This is very plausible since in the D-case the time necessary for obtaining a certain energy is ~~max~~ twice as long as in the normal case, so that the radiation-damping which increases with the time can gain more influence in the D-case.

Applying (5) to a 200KV D-betatron, with a radius of about 70cm ( $B \approx 1$ ) we obtain a minimum frequency of  $450 \text{ sec}^{-1}$  if  $\Delta R$  is chosen 3cm. A normal betatron would have a radius of 140cm if the same alternating magnetic field strength was applied, which would render a minimum frequency of  $150 \text{ sec}^{-1}$  approximately.

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